A 30' by 48' test building was constructed in 2018 to examine the possibility of using this type of construction for housing for those with Electrical HyperSensitivity (EHS) and Multiple Chemical Sensitivity (MCS). It was permitted as a shop (no bathroom, no kitchen, bare concrete floors). The shell of the building was finished in September, 2018. I then installed interior walls of knotty aspen to form two ‘bedrooms’ at one end of the building (tack rooms in a shop). I also built a ‘kitchen’ cabinet base section with a concrete countertop and a sink with hot and cold running water. Transfer of my office and lab from the adjacent house then occurred in early 2019. This document deals with the measured energy efficiency of the building.

The building is a metal skin structure built by Morton Building Company. This is a nation wide company that has been in business for over a century. This type of construction is widely used for shops, barns, garages, and commercial buildings, but in recent years is also used for houses. An inspection of the Morton website will reveal many attractive homes they have built. Their standard practice on insulated buildings is to use the same metal siding on the inside walls and ceiling as on the roof and exterior walls. This makes the building into what might be called a Faraday cage, a double metal wall structure built to keep electromagnetic signals out. The measurement of cell phone signals inside and outside the shop is the subject of another document found on the website www.emsri.org.

The standard Morton package has R-38 insulation in the ceiling, R-19 in the walls, and double glazed windows. The vapor barrier on the walls does not have penetrations for electrical outlets, which should result in the building being relatively tight regarding infiltration and exfiltration. The ceiling vapor barrier has penetrations for the light fixtures, which will cause some exfiltration heat loss. I did not ask for special treatment for this or any other leakage concern, so my measurements are for a ‘standard’ building.

Rockvale, Colorado is located in a relatively mild climate, sometimes referred to as
the ‘banana belt’ of Colorado. The average number of Heating Degree Days is about 5000. We get several snows each winter, and often the ground will be bare again within 24 hours. I will be using the building as a shop/office/lab. If the weather is really bad I can stay at home. An uncomfortably low temperature in the shop is not a significant problem as long as the pipes do not freeze. Therefore I did not spend any ‘extra’ to get a high efficiency structure. But if this is to be a prototype for a house, built either nearby on an empty lot, or elsewhere in the country, we need to know the efficiency. This topic has been of considerable interest in the building trade for over a half century, so one can find much information on the Internet. Better information is available at a good University library in the form of technical papers not available for free on the Internet.

The question most of us want answered is: What will be the yearly operating cost for a given type of heating (and cooling)? This shop is all-electric, like the adjacent house. Electric power comes to a 300 A breaker box on the side of the house, then through a 100 A breaker into a conduit to the shop. I bought an old style analog electric meter on eBay ($28) and had the electricians install the meter between the main breaker box and the shop. I read the analog meter weekly, so I can calculate the total electrical energy used in the shop since the previous reading. I calculate the cost per kWh from the utility bill for the combination of house and shop ($0.1773/kWh for February, 2019). A simple multiplication gives me the cost of heating the shop for that interval.

Not all the electric power is consumed by the baseboard heaters. Some is used by lighting, some by a computer, etc. But except for the photons that escape through the windows, all the input power stays inside the building, and appears as heat. A house has more losses than a shop, of course, such as hot water from shower or washer going down the drain, and moist heated air from a clothes dryer being blown outside the house.

Energy usage for the house and shop is given in Table 1 for 2018-2020. The house is 1500 ft$^2$, three bedroom, and two baths, on an uninsulated slab. It is partially bermed into the hill behind it. The back wall and the two side walls are concrete; the front wall is frame. Electric baseboard heaters supply the heat. Air conditioning is by evaporative cooler driven by two photovoltaic panels and a 24 VDC battery bank, hence does not affect the warm weather utility usage in Table 1. A tenant (my daughter’s boy friend) moved into the house in early 2018. He did not mind a low thermostat setting of 65°F or lower (a good thing) but liked to sleep with a window cracked open (a bad thing in cold weather!).

Another tenant moved to my site 10/11/19, leaving 1/15/20. She was 62 years old, thin, cold blooded, and quite ill with Parkinson’s, Lyme, photosensitivity, and EHS. She lived in the house the first part of the period, then shifted to the shop. This caused
significant increases in energy consumption during that period.

The shop has six 1 kW baseboard heaters, three on a common thermostat on the wall and the other three on individual thermostats on the heaters themselves. The latter three are left completely off when no one else is staying here. The wall thermostat is set on 62°F. I typically keep my cap, sweater, and jacket on during the heating season. Three heaters are quite adequate to maintain this low temperature setting.

The average energy usage during warm weather, mid May to mid October, was about 70 kWh/month. This non-heating portion (lighting, cooking, refrigerator, hot water, computer), would be fairly steady throughout the year. This would total about (12)(70) = 840 kWh/year. The heating bill for the shop for 2020 would then be in the ballpark of (4834 - 840)(0.1773) = $708/year. The heating bill for the house for 2020, using the same technique, would be about $973. The shop heating bill is about 73% of the house heating bill for the circumstances of this one year.

I was curious about the maximum indoor temperature that could be reached with all six heaters. I turned everything off for a few days, allowing the interior temperature to drop to 37°F. I then turned all heaters on. The thermostats would not allow 100% duty cycle, but allowed an input of 691 kWh over the period 1/14/19-1/21/19, an average of 4.11 kW. The indoor air temperature increased to 71°F the second day of the test, but could not increase further. This would represent a monthly bill of over $540, just for shop heating. If a still warmer indoor temperature were desired, more heaters would need to be added.

What can be done to lower heating costs? Which of these improvements have the shortest payback period? These questions require a detailed analysis of heat flows in the building, which the remainder of this document discusses.

Instrumentation

Anticipating these questions, I installed some instrumentation during construction. Morton does not have a crew that pours concrete so I hired a local contractor to pour the foundation. It is called a monolithic pour since the foundation and slab are poured as a single unit. A not-to-scale sketch is shown in Figure 1. The slab is 4 inches thick. The outside edge of the foundation is about 2 feet in depth. The width of the building is 30 feet and the length is 48 feet. There are 2 inches of R-10 insulation against the perimeter of the foundation, all the way around the building. I also had the contractor place 6 mil vapor barrier over the fill before the concrete was poured. This helps to keep the humidity low by preventing moisture in the earth from wicking through the concrete into the interior of the building. There is no insulation under the slab. Neither Morton nor the contractor offered any significant advice about the desirability of insulation under the slab.
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Table 1: Energy usage house and shop, kWh
I bought a spool of Type T thermocouple wire and installed thermocouples at several locations in and under the concrete. $T_1$ is directly below the outside edge of the foundation, 30 inches down from the top of the slab. There are four thermocouples located in and under the slab near the center of the building. $T_{10}$ is 27 inches below the top of the slab, while $T_{11}$ is just under the vapor barrier next to the slab, both installed before the concrete was placed. During early testing I drilled a hole through the slab and installed $T_2$ about 18 inches below the top of the slab and $T_3$ in a saw cut.

I bought an Extech 12 channel thermocouple datalogger ($\$989$) to record the temperature data on an SD card. This was programmed to record all temperatures once per hour. At appropriate intervals the SD card was removed and the data transferred to a computer for analysis. If the card was replaced during the one hour window between data collections, the Extech would just continue appending temperature data to the same file. A spreadsheet program was used to analyze the data. I tried Microsoft Excel first, but soon shifted to LibreOffice Calc, a Linux based application.

Thermocouples $T_4$, $T_5$, and $T_8$ are installed on wood nailers on the side of the building, low, high, and middle heights, close to the interior panels that would be installed later. $T_6$ is on the ceiling, near the center of the building. $T_9$ hangs below the metal enclosure holding the datalogger. The average of these five thermocouples is set equal to $T_{in}$, the interior temperature of the building. There is one thermocouple ($T_7$) outside the building, between the house and shop, about five feet above grade, protected by a plastic shroud. $T_7$ is set equal to $T_{out}$, the exterior temperature of the building, in calculations to follow.

It should be noted that heat flow analysis is well outside my area of training. I was unable to find a paper on a similar research effort that used an electrical model for heat flow that I could understand. The following discussion may be somewhat nonstandard, but it makes sense to me. My electrical model for heat flow in the Morton building is shown in Fig. 2. Temperature causes heat to flow through a thermal resistance, like
voltage causes current to flow through an electrical resistance. A capacitor indicates heat storage, in earth or concrete.

![Simplified electrical model of heat flow in Morton building.](image)

I have thermocouple data for the thermocouples $T_3$, $T_{11}$, and $T_2$ between the inside temperature $T_{in}$ and the temperature $T_{10}$ 27° down, which could be represented by a sequence of series resistors and shunt capacitors in Fig. 2. I looked at these data in some detail but eventually decided to use the simpler model of Fig. 2 for the following reasons:

1. The 24 hour averages for $T_3$ and $T_{11}$ are virtually identical, making the tiny differences not an accurate indicator of heat flow.

2. The $T_2$ thermocouple was noisy. The recorded temperature could easily change by 5 or 6°F from one hour to the next. The 24 hour average was usually within a degree or so of a plausible value, but I still lacked confidence in it providing meaningful results.

Heat flow is often represented by the letter $Q$, often with the units BTU/hr. There are 3410 BTU in 1 kWh, so we can also express heat flow as kWh/hr, or simply a power flow kW. By conservation of energy, we can say that the power entering the building is equal to the power leaving the building, at least averaged over an appropriate time period so storage is not a factor.

$$Q_{in} = Q_{out}$$

$Q_{in}$ consists of electrical power from the utility $Q_u$, electrical power from nominal 1200 W of photovoltaic panels $Q_{pv}$, direct solar gain from light shining through the windows $Q_{gain}$, and $Q_{people}$, the heat given off from the people inside the building (about 400 BTU/hr/person). The last term is important in very high efficiency houses such as Net Zero or PassiveHaus construction. I will ignore it for my case.
\[
Q_{\text{in}} = Q_u + Q_{pv} + Q_{\text{gain}} + Q_{\text{people}} \tag{2}
\]

\(Q_{\text{out}}\) consists of heat flow through the shell of the building (walls, ceiling, windows, and doors), heat flow due to infiltration/exfiltration, and heat flow through the slab.

\[
Q_{\text{out}} = Q_{\text{shell}} + Q_{\text{infil}} + Q_{\text{slab}} \tag{3}
\]

\(Q_{\text{in}}\) is a one-way flow into the building. Heat flows in \(Q_{\text{out}}\), on the other hand, may flow in either direction. If the outdoor temperature is greater than the indoor, outside heat will flow into the building through the shell.

All the heat flows have a diurnal component, so we are never in a true steady state environment. All the thermal mass inside the building will be increasing or decreasing in temperature throughout the day. I have hourly temperature data, but to reduce the effects of storage, it is probably better to use daily average power flows (units kWh/day). A weekly average power (kWh/week) would be even less affected by diurnal power flows in and out of storage.

In addition to the power purchased from the local utility, there is power input from 1200 W (nominal) of solar panels. This is used for two different experiments, one to charge recycled lithium ion batteries to operate the overhead LED lights and the second to directly heat the interior space by operating a bank of incandescent bulbs. There is a day-to-day variation in input power from variation in cloud cover, as well as variation in experimental factors. On a sunny day the net solar PV energy into the building is on the order of 5 kWh. Part of the experiment is to meter this value and record on an SD card, but this has not happened yet. I obviously discontinue the second part of the experiment in warm weather.

**Solar Gain**

The power in the sun’s rays at solar noon is on the order of 1 kW per square meter (perpendicular to the ray) on a clear day. A house might be oriented such that a window is perpendicular to the sun at sunrise or sunset, but normally a window is at an angle to the sun, and the angle varies throughout the day. Incident power declines rapidly with clouds. A value of 1000 W/m² could easily drop to below 50 W/m² under a dark cloud.

One program that calculates the amount of solar energy striking a window is available at pvwatts.nrel.gov. The program asks for the azimuth and tilt of the window (or PV panel), and the location. It predicted an average energy of 4.60 kWh/m² per day for January for zip code 81244 (Rockvale, CO), a tilt of 90° (vertical), and an azimuth


of 146° (generally southeast). The actual energy would vary from less than 1 kWh/m² on a cloudy day to perhaps 6 or 7 kWh/m² on a completely clear day.

Part of the solar power hitting the window is reflected back into space, part is absorbed into the glass, and part is transmitted into the house. The fractions change with coatings applied to the glass, the thickness of the glass, the number of layers, etc. Values of the transmitted fraction seem to be in the range of 0.5 to 0.6 on the Internet. I will select a value of 0.5.

The Morton building has two windows on the southeast and two on the southwest. Each window has a glass area of 0.777 m². The southeast wall has PV panels mounted on the wall above the windows, which shade the windows except for times when the sun is low in the sky. There is still some direct sunlight through the gaps between PV panels. Just for ballpark purposes, I will select a transmittance ratio of 0.2 for the southeast wall. The average daily gain for this wall (azimuth = 146°) is

\[ W_{\text{dailySE}} = 4.60(2)(0.777)(0.2) = 1.43 \text{ kWh/day} \]  

Similarly, the average daily gain for the southwest wall is

\[ W_{\text{dailySW}} = 3.74(2)(0.777)(0.5) = 2.9 \text{ kWh/day} \]

That is, during an average day in an average January, we would expect something like 4.3 kWh to enter the heated space through the windows. Given the crude assumptions made, we can round this off to 4 kWh. The purchased energy for heating is on the order of 800 to 1200 kWh/month (from Table 1) or 27 to 40 kWh/day. Just for illustration, on a cold day in January, \( Q_{\text{in}} \), on average, would be the sum of about 40 kWh from the utility, 5 kWh from the PV panels, and 4 kWh from sunlight through the windows.

**Heat Flow Through Shell**

\( Q_{\text{shell}} \) includes the heat flow out of (or into) the building through the walls, ceiling, windows, and doors.

\[ Q_{\text{shell}} = Q_{\text{walls}} + Q_{\text{ceiling}} + Q_{\text{windows}} + Q_{\text{doors}} \]  

The general form of the heat loss for these items is given by

\[ Q = \frac{A(T_{\text{in}} - T_{\text{out}})}{R} = UA(T_{\text{in}} - T_{\text{out}}) \text{ BTU/hr} \]
where $A$ is the area in ft$^2$ (or m$^2$), $T_{in}$ and $T_{out}$ are the inside and outside air temperature in °F (or °C), $R$ is the ‘resistance’ of the building element to heat flow, with units (hr)(ft$^2$)(°F)/(BTU) (or (°C)(m$^2$))/W), and $U = 1/R$ is the heat transfer coefficient or the heat loss coefficient. This equation is valid only under steady state conditions, constant inside and outside temperatures, no wind, etc.

The inside dimensions of the building are about 28.5’ wide by 46.5’ long by 10’ high. The ceiling area is $A_c = 1325$ ft$^2$ (≈ 125 m$^2$). The total wall area is 1500 ft$^2$. There are two outside doors of size 41” by 82”, giving a total area for doors $A_d = 46.7$ ft$^2$. There are 7 windows of size 52” by 33”, giving a total area for windows of $A_{wi} = 83.4$ ft$^2$. Subtracting these two numbers from the gross wall area gives a net wall area of $A_{wa} = 1370$ ft$^2$. The building volume is $(1325)(10) = 13,250$ ft$^3$.

Morton provided me an Envelope Compliance Certificate with R and U values that can be used for heat loss analysis. There was one column for “Proposed U-Factor” and another column for “Budget U-Factor”. The budget U-factor values were equal to or larger than the proposed U-factors. I assume the proposed U-factors are the numbers provided by the various manufacturers and the budget U-factors represent an estimate of how good the installed component is likely to be. For example, the wall has R-19 fiberglass insulation. There are air spaces between the fiberglass and the metal panels that have R-values of 0.91 each. There are air films on the two sides of the assembly that add a total R-value of 0.85. Part of the wall consists of wood framing rather than insulation, with a different R-value. The process is like calculating the equivalent resistance of two parallel resistors. Morton calculated a total assembly R-Value of 21.311. I assume this is for the portion of the wall containing insulation. The U-value is $1/21.311 = 0.047$. They then list a budget U-value of 0.064, (R-Value of 15.6) a value 36% higher. I assume this is for the entire wall, where part of the heat flows through the wood columns. They also list a budget U-value for windows of 0.35 and a budget U-value for doors of 0.7.

The various heat flows can be written

\[ Q_{ceiling} = \frac{1325}{38}(T_{in} - T_{out}) = 34.9(T_{in} - T_{out}) \]  \hspace{1cm} (8)

\[ Q_{walls} = (0.064)(1370)(T_{in} - T_{out}) = 87.7(T_{in} - T_{out}) \]  \hspace{1cm} (9)

\[ Q_{windows} = (0.35)(83.4)(T_{in} - T_{out}) = 29.2(T_{in} - T_{out}) \]  \hspace{1cm} (10)
\[ Q_{\text{doors}} = (0.7)(46.7)(T_{\text{in}} - T_{\text{out}}) = 32.7(T_{\text{in}} - T_{\text{out}}) \]  

(11)

The combined heat flow for the walls, ceiling, windows, and doors is

\[ Q_{\text{shell}} = 184.5(T_{\text{in}} - T_{\text{out}}) \quad \text{BTU/hr} \]  

(12)

\[ Q_{\text{shell}} = 0.054(T_{\text{in}} - T_{\text{out}}) \quad \text{kWh/hr} \]  

(13)

\[ Q_{\text{shell}} = 1.3(T_{\text{in}} - T_{\text{out}}) \quad \text{kWh/day} \]  

(14)

where \( T_{\text{in}} \) and \( T_{\text{out}} \) are in degrees Fahrenheit.

**Infiltration**

We now need to add a term for infiltration, which will vary widely depending on the quality of building construction. From my Internet search, a well constructed building may have only 0.35 or 0.4 Air Changes per Hour (ACH). Typical buildings may have ACH values around 2.0. I found an equation on the Internet for \( Q_{\text{infil}} \),

\[ Q_{\text{infil}} = (0.018)(ACH)(Vol)(T_{\text{in}} - T_{\text{out}}) \]  

(15)

which, for an ACH of 0.35, yields

\[ Q_{\text{infil}} = (0.018)(0.35)(13250)(T_{\text{in}} - T_{\text{out}}) = 83.5(T_{\text{in}} - T_{\text{out}}) \quad \text{BTU/hr} \]  

(16)

or

\[ Q_{\text{infil}} = 0.6(T_{\text{in}} - T_{\text{out}}) \quad \text{kWh/day} \]  

(17)

or slightly less than half the value for \( Q_{\text{shell}} \).

My Morton building has no stovepipe and no exhaust fan. Infiltration is therefore mostly a function of wind speed. Wind will blow outside air into the building through the cracks on the windward side and suck inside air out on the leeward side. I suspect that during calm conditions that \( Q_{\text{infil}} \) will be close to zero.

Actually, the air pressure on a wall perpendicular to the flow of wind is proportional to the square of the wind speed. That suggests that \( Q_{\text{infil}} \) for a 20 mph wind may be on the order of four times that for a 10 mph wind.

The ‘leakiness’ of a building can be estimated by a blower door test, where an instrumented fan is placed in a door, and the remainder of the opening is filled in with
an impermeable membrane. The fan speed is adjusted until the differential pressure across the fan is 50 Pascals, and the quantity of air passing through the fan is measured. Contractors will use this test to find and caulk cracks in the building structure.

Suppose I do a blower door test and am able to get the ACH down to 0.4 at 50 Pascals. What does that mean in terms of day-to-day losses in real life conditions? I have not found an understandable answer to that question. It appears to me that $Q_{infil}$ can vary from less than 10% of $Q_{shell}$ in near-calm conditions to perhaps even more than $Q_{shell}$ during windy periods. I am not even sure how $Q_{infil}$ would be measured. We might build a house, well insulated from earth, with very little thermal mass inside, place a differential pressure meter on each outside wall, install at least two anemometers on opposite sides of the building, and record lots of data (every few seconds rather than every hour). It is conceivable that one could sniff out the variation of $Q_{infil}$ with respect to wind speed from processing enough data over a sufficient period of time. At the moment, I am inclined to leave that particular research project to someone else.

**Heat Flow Through Slab**

An equation for the heat loss through the slab, using the model in Fig. 2, is given by

$$Q_{slab} = \frac{A(T_{in} - T_{10})}{R_1} = \frac{A(T_{10} - T_{earth})}{R_2} \text{ BTU/hr} \quad (18)$$

in steady state ($T_{in}$, $T_{10}$, and $T_{earth}$ constant for a sufficient period of time that $C_1$ is not charging or discharging).

At a sufficiently great depth, the temperature of the earth, $T_{earth}$ will be constant year around. This is usually very close to the average air temperature, around 52°F in Rockvale. If the air temperature is above this value, heat will flow into the earth. If below this value, heat flows out of the earth to heat the air. At depths of a few feet, the temperature will have a yearly cycle similar to the yearly air temperature, but with a lower amplitude and with a lag related to the time required for heat energy to propagate through the earth.

For my data set, $T_{10}$ was always greater than 52°F, my assumed value for $T_{earth}$. That means that heat flow will always be to the right through $R_2$. However, heat flow can be either to the right or to the left in $R_1$. It will be generally to the right in the warm months, helping to cool the building, and generally to the left in cold months, helping to heat the building.

**Data Analysis**

My data set was for the period 2/5/19 to 1/18/21, with a few gaps. I used LibreOffice
Calc to find the average of adjacent 24 hour periods. I have a data set of 647 days.

Figure 3 shows the temperature variation of the buried thermocouples. $T_1$, 30” down at the edge of the building, varies from about 45°F to about 73°F, with an average value of 58.7°F. The average outdoor air temperature for the same period was 55.2°F, and the average indoor temperature $T_{in}$ was 65.3°F. We see that the building, 10.1°F warmer than the outside on average, raised the temperature under the perimeter of the building by 3.5°F. This increase in temperature represents a flow of energy into the earth, partly from building overheating in the summer (free) and partly from operating electric heaters in the winter (expensive).

![Figure 3: T1 edge of building 30” down, T3 in slab saw cut, T11 bottom of slab, T10 is 27” down, center of building.](image)

The effect of energy storage under the slab is quite obvious from this figure. During the heating season, the temperature under the slab drops only to about 60°F, while the temperature around the perimeter of the building drops to about 45°F. Further away from the building, the temperature two feet down would drop to perhaps 32°F.

Figure 4 shows a similar plot for $T_{in} - T_{10}$.

I have experimental values for $T_{in} - T_{out}$ and $T_{in} - T_{10}$, but am unsure on what coefficients should be used to satisfy the requirement that power in must be equal to power out. Experimentally, there will always be some difference between the two. This error can be written as
Figure 4: Difference between indoor air temperature $T_{in}$ and temperature of earth 27" below top of slab $T_{10}$

$$\text{Error} = Q_p + Q_{gain} - Q_{shell} - Q_{infil} - Q_{slab}$$

(19)

where $Q_p$ includes power from both the electric utility and the PV panels. From the model in Fig. 2, $Q_{shell}$ and $Q_{infil}$ are both proportional to $T_{in} - T_{out}$ while $Q_{slab}$ is proportional to $T_{in} - T_{10}$. We can rewrite the above equation as

$$\text{Error} = Q_{in} - 1.3(T_{in} - T_{out}) - B_1(T_{in} - T_{out}) - B_2(T_{in} - T_{10})$$

(20)

where $B_1$ and $B_2$ are constants to be determined. Note that $B_1$ is not actually ‘constant’ since the infiltration loss is proportional to both the temperature difference from inside to outside and the air pressure difference across the wall, which in turn is proportional to the square of the wind speed.

A brute force method of solution is to iterate values for $B_1$ and $B_2$ and sum the daily errors over the length of the data set. When this sum is minimum, we have an
<table>
<thead>
<tr>
<th>Days</th>
<th>Dates</th>
<th>$Q_{in}$</th>
<th>$Q_{shell}$</th>
<th>$Q_{infil}$</th>
<th>$Q_{slab}$</th>
<th>Net</th>
</tr>
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<tr>
<td>2-71</td>
<td>2/5/19-5/24/19</td>
<td>1243</td>
<td>1214</td>
<td>393</td>
<td>-86</td>
<td>-278</td>
</tr>
<tr>
<td>72-175</td>
<td>5/25/10-10/2/19</td>
<td>665</td>
<td>-118</td>
<td>-38</td>
<td>1219</td>
<td>-398</td>
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<tr>
<td>176-373</td>
<td>10/3/19-4/17/20</td>
<td>5953</td>
<td>4931</td>
<td>1596</td>
<td>-1435</td>
<td>861</td>
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<tr>
<td>374-516</td>
<td>4/18/20-9/7/20</td>
<td>1208</td>
<td>53</td>
<td>17</td>
<td>1521</td>
<td>-383</td>
</tr>
<tr>
<td>517-648</td>
<td>9/8/20-1/17/21</td>
<td>2399</td>
<td>2370</td>
<td>767</td>
<td>-989</td>
<td>251</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>11468</td>
<td>8450</td>
<td>2735</td>
<td>230</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 2: Seasonal energy flows in Morton building in kWh.

To find $B_1$, I looked at the 69 days (of 647 total) where $T_{in} - T_{10}$ was less than a one degree difference (either positive or negative), thus having little effect on calculating the error. This gave $B_1 = 0.42$. This is not drastically different from the Internet value of 0.6, shown in Eq. 17. A value of $B_2 = 2$ then gave the minimum error for the entire data set.

One type of validity check is to see if the predicted error values track the calculated values. I found the average daily error for five ranges of $T_{in} - T_{out}$, less than zero, 0 - 10, 10 - 20, 20 - 30, and greater than 30°F. The results were +0.32, -0.68, +2.78, +0.34, and -7.8 kWh/day. The predicted average daily error agrees with the measured fairly well except not as well on the extremely cold days. $Q_{in}$ on these cold days is on the order of 50 kWh/day, so an error value of -7.8 kWh/day only represents a 15% error.

I now want to discuss the seasonal energy storage aspects of the slab and earth under the slab. During the heating season the slab is helping to heat the shop, and the opposite during the cooling season, with opposite directions of heat flow. I could have made some arbitrary selection of dates for the two seasons, but instead I chose to look at the variation of $T_{in} - T_{10}$, shown in Fig. 4.

There are two distinct humps in this figure where $T_{in} - T_{10}$ is well above zero. This is in the spring and summer when the earth is recharging its thermal energy. I defined the cooling season to begin when $T_{in} - T_{10}$ went positive (and mostly stayed positive), and to end when it went negative. This gave me three heating seasons and two cooling seasons for my data set. I then summed the daily energy flows over the entire season. The results are shown in Table 2.

$Q_{shell}$ and $Q_{infil}$ show as negative values during the first cooling period, Days 72-175. The average outside temperature was greater than the inside, so heat was flowing into the building. $Q_{slab}$ could be considered the net energy stored in the earth for that period, 1219 kWh. During the following heating period the heat flow out of the earth
was 1435 kWh. This was the one full heating season of the data set. This augmented the \( Q_{in} \) of 5953 kWh. At 0.1773 cents/kWh, this 1435 kWh from the earth saved me $254.

The total of the \( Q_{slab} \) values over the entire data set was a positive (into the earth) 230 kWh, as it must be when the average indoor temperature is greater than the temperature in the earth.

The accumulative errors in the last column consist of three negative and two positive values, which add to a net error over the entire data set of only 53 kWh, suggesting the results may at least somewhat represent reality.

**Thermostat At 70°F**

We have about two years of measured data on energy consumption of the Morton building with the thermostat set at about 60°F. There is no air conditioning and \( T_{in} \) will rise to about 78°F in the summer. Let us now try to estimate energy consumption with the thermostat set at 70°F all year. The temperature profile of the slab and earth under the building should reach a quasi constant condition, where storage is full. No energy will be flowing in or out of \( C_1 \), so we can ignore it.

We need to find periods in our data set where \( T_{in} \) and \( T_{10} \) are almost constant for long periods of time. Plots of these two temperatures are given in Fig. 5 and Fig. 6.

I picked the least variable periods in Fig. 5 as the days when \( T_{in} \) was greater than 75°F, days 125-175 and 445-520. Likewise I picked the least variable periods in Fig. 6 as the days when \( T_{10} \) was greater than 69°F, days 132-188 and 464-560. The average \( T_{in} \) for the two periods were 75.64°F and 75.82°F. The average \( T_{10} \) values were 70.57°F and 70.79°F, almost exactly a 5°F difference. Assuming that \( B_2 = 2 \), we have \( Q_{slab} = B_2(T_{in} - T_{10}) = 2(5) = 10 \text{ kWh/day (1420 BTU/hr)} \) flowing to earth. From Eq. 18 the R-value for the slab plus two feet of soil is

\[
R_1 = \frac{(1325)(5)}{1420} = 4.66 \quad (21)
\]

Since storage is full, the same 10 kWh/day is flowing through \( R_2 \) to \( T_{earth} \). We assume \( T_{10} = 70 \) and \( T_{earth} = 52 \) and find

\[
R_2 = \frac{(1325)(70 - 52)}{1420} = 16.8 \quad (22)
\]

which makes the total R-value of slab plus earth equal to \( R_1 + R_2 = 4.66 + 16.8 = 21.46 \). This is far too precise, given the many assumptions and approximations made. We should round off to \( R = 21 \), suggesting that we think the actual result might be between
Figure 5: Temperature $T_{in}$

R = 20 and R = 22, or even to R = 20 which might suggest we think truth lies between R = 15 and R = 25. I will use R = 21, but acknowledge that reality might well lie outside R = 20 and R = 22.

When the thermostat is set to a constant 70°F for the whole year, the heat flow through the slab will be

$$Q_{slab} = \frac{(1325)(70 - 52)}{21} = 1136 \text{ BTU/hr} = 8 \text{ kWh/day} \quad (23)$$

Consider the case where $T_{out} = 70°F$, and we can assume $Q_{in} = 0$ (cloudy period, lights off over the weekend, etc.) What happens to the indoor temperature? The earth is still sucking 8 kWh/day from the interior space. The indoor temperature must drop below 70°F until $Q_{shell} + Q_{infil} = \text{the heat flow into the earth.}$
which results in $T_{in} = 65.35^\circ F$ for $T_{out} = 70^\circ F$. Most of us would open a window or put on a sweater rather than turn on the heat under these circumstances, but one or the other has to happen. Either we endure chilly temperatures or we buy electricity to heat the earth.

We are now able to make an informed estimate of the extra energy required to raise the thermostat from 60°F to 70°F for a one year period. We choose the period 7/1/19-6/30/20 and ask LibreOffice Calc to find the sum of $Q_{shell} + Q_{infil} + Q_{slab}$ for the two thermostat settings AND just for the days when $T_{out}$ is less than 70°F. For the thermostat setting of 60°F we calculate $(1.3 + 0.42)(T_{in} - T_{out}) + 2(T_{in} - T_{10})$, with the result 7027 - 1179 = 5848 kWh. For the thermostat setting of 70°F we calculate $(1.3 + 0.42)(70 - T_{out}) + 8$, with the result 10410 + 1976 = 12386 kWh. $Q_{shell} + Q_{infil}$ increased
by 10410 - 7027 = 3383 kWh while $Q_{slab}$ increased by 1976 - (-1179) = 3155 kWh. At $0.1773$/kWh the yearly heating cost increased from $1037$ to $2196$, an increase of $1159$.

**Conclusions**

The Morton building used a total metered electricity of 4658 kWh in 2019 and 4834 kWh in 2020, at costs of $825.86$ and $857.07$ per year for an electricity cost of $0.1773$/kWh. Interior temperature varied from about 60°F in cold weather to about 78°F in hot weather. I calculated in the previous section that changing the thermostat to 70°F would increase the electric bill by $1159$/year, making the total electric cost for the building somewhere around $2000$/year.

By way of comparison, my wife and I live in a three bedroom, two bath, 1500 ft² house in a nearby town. We never open a window (allergies). We use regular air conditioning rather than a swamp cooler. The thermostat is set on 72°F in the winter and 74°F in the summer. Total energy cost for 2020 was about $1000$ for electricity and
about $500 for natural gas. Natural gas is cheaper than electricity for heating so this is not a fair apples-to-apples comparison, but still gives a rough comparison.

One of my many questions about the Morton building is whether I should have insulated the slab. My concrete contractor tells me that he never insulates the slab except for the case where in-floor heating is used, and even then uses only R-10 insulation. My first reaction was that this is a bad habit, dating back to the days of cheap energy. Now I have some numbers to check my gut reaction.

The floor is cold in the building all year. A cold floor is a disadvantage in the winter time (cold feet) but an advantage in the summer time. I leave the windows and doors closed in the summer and get by with just a fan (no swamp cooler). Adding insulation under the slab will make it essential to install some sort of air conditioning in the building.

Home Depot sells R-20 foam insulation (actually two R-10 boards stacked together) for $1.81/ft$^2$. If I had installed R-20 under the concrete, I would now have a total of R-41 (R-21 for the earth and R-20 for the foam). Double the insulation would cut $Q_{slab}$ in half, from 1976 to 988 kWh for the year. The cost is $(0.1773)(988) = $175.17 for the year, or $(175.17)/(1325) = $0.132/ft^2$. The simple payback is $1.81/$0.132 = 13.7 years.

This is a rather long payback, especially since it is just for the materials. The payback gets longer when one includes labor for installation, and for what possibly becomes a required air conditioner. I hate to admit it, but it appears my gut feeling was wrong. Standard practice of not insulating under the slab makes reasonable economic sense.

But comfort is also an important factor. In my case, a nearly constant temperature in the range of 70-74°F for a house is very important. The cost of extra insulation is a small percentage of the cost of the house. The payback period may be longer than the economically desirable 5-10 years, but is offset by better comfort.

The following table shows the predicted energy loss for this building through the ceiling, walls, windows, doors, and slab for 7/1/19–6/30/20 for $T_{in}$ held at 70°F and the slab with R20 insulation under it. These are the building components which depend on R-value. My estimate for infiltration, which depends more on quality of construction, is also included. The total energy loss through these components is 11,390 kWh ($2019.45 at current electricity prices). I am sure that a large fraction of our population is paying similar heating costs on a per square foot basis, and would consider this level of energy usage acceptable. But are there any options for decreasing energy usage?

Attic insulation is not terribly expensive nor hard to install. Two kids showed up
\[\begin{align*}
Q_{\text{ceiling}} &= 1487 \\
Q_{\text{walls}} &= 3736 \\
Q_{\text{windows}} &= 1244 \\
Q_{\text{doors}} &= 1393 \\
Q_{\text{infil}} &= 2542 \\
Q_{\text{slab}} &= 988 \\
\text{Total} &= 11390
\end{align*}\]

Table 3: 2019-2020 predicted energy loss from Morton building in kWh.

at noon, ran the hoses, blew in the specified depth of insulation, and were gone by 2 pm. Morton subcontracts this activity. They specify R-38, probably out of habit. It is not a problem to ask for deeper insulation, at a nominal price increase. I will definitely inquire about using R-60 or even higher in any future building. At R-60 we should reduce the ceiling loss by about a third, or perhaps 500 kWh/year.

The fiberglass insulation in the walls is rated at R-19. Morton suggests using a value of R-15.6 (U = 0.064) for the composite wall structure. Offhand I do not see a simple method of significantly increasing R without drastically increasing construction cost. One idea I considered was adding 1.5 inches of foam board between the exterior nailers before the exterior steel panels are installed. The R-value of this thickness of foam is about 7.5. It would be in a thermal parallel with the nailers, so the R-value increase of the wall assembly would be in the range of R-6 to R-6.5, making the final R-value about R-22. This would reduce the wall loss by perhaps 1000 kWh/year.

The external nailers on my building are spaced 31” apart on center. Therefore the foam inserts need to be 27.5” wide. If I split a 4’ by 8’ foam board, I have a piece 20.5” wide, not wide enough for the next section. I can cut 7” off another foam board and splice the two pieces together. This idea is beginning to look like it involves either substantial waste, or extra time for installation.

Morton seems flexible about sourcing of windows and doors. I let them supply both on this building, but will look into higher efficiency components to buy myself for any future building. In particular, I will take a look at fiberglass windows rather than the vinyl windows in this building. One Internet site said this was a preferred efficiency upgrade in Canada. I also like the idea of the twist tilt windows used in Europe. They appear to have better sealing through stronger mechanical construction. They also tilt into the room, thus not interfering with a metal screen across the entire window opening on the outside of the building.

We could also reduce the number of windows, an idea I do not like because I like as
much exterior light on the inside of the building as possible. We could also reduce the
number of doors from two to one. The house on the property is bermed into the hill
behind it, such that one door makes perfect sense.

Reducing $Q_{infil}$ is very important to contractors today. Tradesmen will go to ex-
treme measures to tape all the seams and spray foam around all openings. The problem
here is that stale air, containing all sorts of smells, pollutants, and toxins, gets stuck
inside the house. This Morton building has bare concrete floors, powder coated steel
panels on walls and ceiling, and bare wood interior walls (no paint, no plywood, etc.),
so it should work with minimal ventilation. One pollutant I did not consider is carbon
dioxide. I bought an Extech SD800 CO$_2$ datalogger to monitor the levels in the build-
ing. After a weekend with no one in the building the meter reads about 350 parts per
million. At the end of an eight hour period the reading may be in the range of 650 to
700 parts per million. It will be even higher at my house in town. With three people in
the house overnight, the reading will be in the range of 800 to 1000 parts per million.

One of the first effects of excess CO$_2$ is sleepiness. Then comes headaches. The level
at which a given person will be affected can vary widely, but one can find expressions
of concern starting on the Internet at about the 1000 parts per million range. It seems
the effect is significant to a good fraction of the population when the level reaches 5000
parts per million.

Modern construction deals with this issue by incorporating a Heat Recovery Ventil-
ator (HRV) or Energy Recovery Ventilator (ERV) into the house. This is a box a few
cubic feet in size with four ports that accommodate flexible duct work, two fans, and
an air-to-air heat exchanger. One fan sucks in fresh outside air and blows it through
the heat exchanger and into the inside air space. The other fan sucks stale inside air
through the heat exchanger and blows it outside. The intent is to adjust the speed of
the two fans until there is no differential pressure from inside to outside, and until the
indoor air quality is acceptable. I see claims of 75% heat recovery for these units.

This will hopefully be the topic of another research project on another test building
at some point. In the meantime I am not confident that we can (or should) reduce $Q_{infil}$
by large fractions over present construction. A reduction of perhaps 500 kWh/year
might be possible.

If we stick with the low hanging fruit of energy efficiency, more insulation in the
attic, 4" foam under the slab, incremental improvements in windows and doors, and
careful sealing of cracks, we should get the heating and ventilating requirement for
this size Morton building into the range of 9000 kWh/year. If this is unacceptable for
some reason, then we need to consider some other construction technique that has a
significantly higher R-value in the walls.